Evanescent Magnetic Field Effect on Transient State of Natural Convection

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Abstract – This paper numerically investigates the effect of an externally evanescent magnetic field on flow patterns and heat transfer of fluid in a square cavity. The horizontal walls of the enclosure are assumed to be insulated while the vertical walls are kept isothermal. A control volume finite element method is used to solve the conservation equations. The effect of constant Hartmann number on Nusselt number was studied. Validation tests with existing data demonstrate the aptitude of the present method to produce accurate results. The effects of inclination magnetic field angle from 0° to 90° on streamlines distributions are shown for different value of Hartmann number. For Grashof number equal to 10^5, the values of relaxation time of the magnetic field are chosen, so that the Lorentz force acts only during the startup transient of the natural convection flow. The Nusselt number was calculated for different values of the inverse relaxation time varying from 0 to +∞. The magnitude and the number of oscillations of the Nusselt number were observed. It has been found that no oscillation was seen at relaxation time equal to 20. Stream lines maps are plotted for different values of dimensionless time. The effect of relaxation time on the transition from single-cell to double-cell configuration was observed. Copyright © 2010 Praise Worthy Prize S.r.l. - All rights reserved.

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I. Introduction

Coupled heat transfer and magnetic field by natural convection in a fluid in square or rectangular enclosures has received considerable attention in the recent years, due to many important engineering applications. The use of an external magnetic field has increasing applications in material manufacturing industry as a control mechanism since the Lorentz force suppresses the
convection currents by reducing the velocities. The study of Oreper and Szekely [1] shows that the magnetic field suppresses the natural convection currents and that the magnetic field strength is one of the most important factors for crystal formation. Ozoe and Marue [2] numerically investigated the natural convection of a low Prandtl number fluid in the presence of a magnetic field and obtained correlations for the Nusselt number in terms of Rayleigh, Prandtl and Hartmann numbers. The effects of magnetic direction in three-dimensional were examined by Ozoe and Okada [3]. It was found that the optimal suppression of the convection currents occurs when the external magnetic field is parallel to the direction of the heat flow. Wakayama and Co-workers [4]-[6] have been active in finding new and notable effects of a strong magnetic field in fluid convection. Garandet et al. [7] proposed an analytical solution to the governing equations of magneto-hydro-dynamics to be used to model the effect of a transverse magnetic field on natural convection in a two-dimensional cavity. Rudraiah et al. [8] used modified alternating direction implicit (ADI) finite difference scheme to solve the vorticity-stream function formulation of natural convection inside a rectangular enclosure in the presence of a magnetic field. The numerical results showed that the magnetic field suppresses the rate of convective heat. Al Chaar et al. [9] numerically studied two-dimensional natural convection in a shallow cavity heated from below in the presence of inclined magnetic field. The numerical results showed that magnetic field reduces the heat transfer and inhibit the onset of the convection current. Furthermore, the convection modes inside the cavity are found to depend strongly upon both the strength and orientation of the magnetic field. A horizontal magnetic field is found to be the most effective in suppressing the convective flow. Al Chaar et al. [10] also investigated the effect of the magnetic drag on the convection currents in vertical differentially heated shallow cavity. The results compared well with a closed form solution obtained by Garandet et al. [7]. Vasseur et al. [11] analytically studied as well as numerically the effect of transverse magnetic field on buoyancy-driven convection in an inclined two-dimensional tall cavity.

Al Najem et al. [12] determined the flow and temperature fields under a transverse magnetic field in a tilted square enclosure. Results showed that the magnetic field on convection currents and heat transfer is more significant for low inclination angles and high Grashof numbers. Yang et al. [13] numerically investigated the effect of the magnetic quadrupole field on a gas natural convection flow.

The results showed that the magnetic force acting on the air in a magnetic quadrupole field exhibits a centrifugal character. Hossain et al. [14] numerically investigated the effect of surface tension on unsteady natural convection flow of an electrically conducting fluid in a rectangular enclosure under an externally imposed magnetic field with internal heat generation. The results showed that a change of direction of the external magnetic field force from horizontal to vertical induces the decrease of the flow rates in both the primary and secondary cells and causes an increase in the effect of the thermo-capillary force.

An increasing in the value of the heat generation parameter leads to the increase in the flow rates in the primary cell well an increase in its size until it occupies almost all of the total cavity space. Cem Ece et al. [15] studied a laminar natural convection flow in the presence of a magnetic field in an inclined rectangular enclosure. The results showed that the flow characteristics and therefore convection heat transfer depend strongly upon the strength and direction of magnetic field, the aspect ratio and the inclination of the enclosure.

The local Nusselt number increases considerably with Grashof number since the circulation becomes stronger and the magnetic field significantly reduces the local Nusselt number by suppressing the convection currents.

The present study is a parametric study and can be considered as extension of previous works in term of the choice for Prandtl number set to be $Pr = 0.71$. Thus, Chaudhary and Jain [16] analytically studied the transient hydromagnetic and thermal behaviour of free convection flow assuming an electrically conducting fluid. For physical meaning of the problem, values of Prandtl number are chosen as 0.71 (air), 1 (electrolytic solution) and 7 (water). It was found that the increase of Prandtl number induces the decrease of temperature. Magnetic field causes the decrease of velocity for both air and water.

Teamah [17] numerically studied steady heat and mass transfer by natural convection flow of a heat generating fluid in presence of a transverse magnetic field in a rectangular enclosure at fixed values of aspect ratio ($A = 2$). Lewis number ($Le = 1$) and Prandtl number ($Pr = 0.7$).

It was found that magnetic field tends to reduce heat transfer and fluid circulation within the enclosure. For Hartmann number $Ha > 20$, average Nusselt and Sherwood numbers have constant values over a range of thermal Rayleigh number, this range increases with increasing Hartmann number. Ishak et al. [18] numerically studied a steady two-dimensional flow of an electrically conducting fluid due to a stretching cylindrical tube. They showed that Nusselt number increases as Prandtl number increases (from $Pr = 0.7$ to $Pr = 7$). Moreover, the effect of magnetic field is found to be more pronounced for fluids with smaller Prandtl number ($Pr = 0.7$), since fluids with smaller Prandtl number have larger thermal diffusivity.

As a consequence and following the above works, the main objective of the present paper is to study the magnetic field effect on Nusselt number evolution in transient heat transfer, at Prandtl number 0.71, without disturbing the steady state. This requires the employment of an evanescent magnetic field, which has not been yet considered.
II. Mathematical Formulation

Imposed magnetic field acting on Newtonian fluid enclosed in a confined differential heated square cavity is considered in this problem, as seen in figure 1. The fluid is modelled as a Boussinesq incompressible fluid, whose properties are described by its kinematic viscosity, thermal diffusivity and thermal volumetric expansion coefficient. The orientation of the magnetic field takes an angle $\alpha$ with horizontal axis. The electric current density vector $\vec{J}$ is defined by:

$$\vec{J} = \sigma \left( \vec{E} + \vec{\omega} \times \vec{B} \right)$$  \hspace{1cm} (1)

The electric force per unit charge (E) is negligible compared to the magnetic force per unit charge (w $\times$ B) as it is given in Woods [19]. Under the above assumptions, the conservation equations for mass, momentum and energy in a two-non-dimensional form are as follows:

$$\frac{\partial U}{\partial \tau} + \text{div} \left( UP - \text{grad}U \right) = -\frac{\partial P}{\partial X} + Ha^2 \left( V \sin \alpha \cos \alpha - U \sin^2 \alpha \right)$$  \hspace{1cm} (2)

$$\frac{\partial V}{\partial \tau} + \text{div} \left( VP - \text{grad}V \right) = -\frac{\partial P}{\partial Y} + \left[ Gr \theta \right] + Ha^2 \left( U \cos \alpha \sin \alpha - V \cos^2 \alpha \right)$$ \hspace{1cm} (3)

$$\frac{\partial \theta}{\partial \tau} + \text{div} \left( \theta P - \frac{1}{Pr} \text{grad}\theta \right) = 0$$  \hspace{1cm} (4)

Where the dimensionless variables are defined by (6):

$$\begin{align*}
Y &= \frac{\nu}{L} \hspace{0.5cm} U = \frac{UL}{V} \hspace{0.5cm} V = \frac{VL}{V} \hspace{0.5cm} \theta = \frac{T - T_e}{T_h - T_e} \hspace{0.5cm} P = \frac{pL^2}{\rho V^2} \hspace{0.5cm} \\
Gr &= \frac{g \beta \Delta T L^3}{\nu^2} \hspace{0.5cm} \tau = \frac{\nu t}{L^2} \hspace{0.5cm} Ha^2 = \frac{B^2 L^2 \sigma}{\mu}
\end{align*}$$

The boundary conditions are:

- $U = V = 0$ for all walls;
- $\theta = 0.5$ on plane $X = 0$ and $\theta = -0.5$ on plane $X = 1$;
- $\partial \theta/\partial Y = 0$ on planes $Y = 1$ and $Y = 0$.

The initial conditions are:

At $\tau = 0$; $U = V = P = 0$ and $\theta = 0.5 - X$ for whole space. The average Nusselt number is given by:

$$Nu = \int_0^L \frac{\partial \theta}{\partial Y} dx$$  \hspace{1cm} (7)

![Fig. 1. Schematic confined diagram of the problem under consideration](image-url)
and $10^5$ respectively is sufficiently enough to carry out this study. The transient study is carried out with a step time $\Delta \tau = 10^{-4}$ for all considered Grashof numbers.

Fig. 2. Nusselt number distribution versus grid size for $Ha = 0$ and $\alpha = 0^\circ$

IV. Results and Discussion

The present study is restricted to the non-reactive fluids with Prandlt number equal to 0.71. The Grashof number ranges between $10^3$ and $10^5$. Our objective is mainly focused on the effect of the magnetic field within the transient state of natural convection, precisely on the fluctuation of the Nusselt number for high Grashof numbers, without disturbing the stationary state. Then, we need an evanescent magnetic field, which can be written as:

$$B = B_0 e^{-\gamma t} \quad (\forall t \in \mathbb{R}^+)$$

Using Eq. (6), the Hartmann number is a decreasing function versus time, and can be written as:

$$Ha = Ha_0 e^{-\left(\frac{\gamma L^2}{v}\right)t}$$

The parameter $\gamma$ was selected so the inverse of the magnetic field relaxation time $n = \left(\frac{\gamma L^2}{v}\right)$ takes prime numbers. Therefore, the Hartmann number can be written as:

$$Ha = Ha_0 e^{-n t}$$

It’s important to notice that for $n$ equal to zero, the magnetic field takes constant value and can therefore disturb the stationary state. The values of parameter $n$ are chosen, so that the magnetic field acts only in the transient state of natural convection for Grashof number equal to $10^5$. This is illustrated in figure 3 which shows the variation of the Nusselt and Hartmann numbers versus time ($Gr = 10^5$). It can be concluded from figure 3 that for $n \geq 20$ the magnetic field affects only the transient regime. For validation purpose, lets begin with the case of $n = 0$ (Constant magnetic field). In this case the Hartmann number, the inclination angle of magnetic field, and the Grashof number range from 0 to 100, 0 to 90°, and $10^3$ to $10^5$ respectively. Figure 4 shows the variation of the Nusselt number versus the Hartmann number at zero inclination angle of magnetic field. Good agreement with the work of Al-Najem and al. [12] is seen from this figure, since heat transfer decreases by increasing the Hartmann number.

Fig. 3. Hartmann number for different constant $n$ and Nusselt number distribution for $Ha = 0$, $Gr = 10^5$ and $\alpha = 0^\circ$ as all function of dimensionless time

Fig. 4. Nusselt number distribution versus Hartmann number for $\alpha = 0^\circ$

Therefore the magnetic field seems to suppress convection and to retards fluid motion via the Lorentz force. Furthermore, for constant magnetic field ($n = 0$), the influence of Hartmann number on the flow patterns was investigated. We plot in figure 5 the distribution of streamlines with the Hartmann number and the inclination angle of magnetic field (for brevity the Grashof number is equal to $10^4$). It can be seen from figure 5, that for small value of Hartmann number ($Ha = 10$), the flow patterns seems to be similar when the inclination angle of the magnetic field increases from 0° to 90°. Whereas, for high value of Hartmann number the flow patterns change noticeably, and one can observe an elongation of the eddy and a clockwise rotation of its
axis from the vertical to the horizontal, when increasing the inclination angle of the magnetic field from 0° to 90°. As can be seen from figure 5, for constant values of magnetic field inclination angle, the increase of Hartmann number tends to slow down the movement of the fluid. Also, the single eddy is observed to be elongates (vertically for 0° and horizontally for 90°). On the other hand, for high value of Hartmann number \((Ha = 50)\), the increase of magnetic field inclination angle, tends to rotate the single eddy axis in clock wise way. In fact, the magnetic field applied in the \(XY\)-plane, generates a magnetic force (Lorentz force). The direction and the magnitude of the Lorentz force are at the origin of the elongation and the axis rotation of the central eddy.

Let's consider the two particular cases of the magnetic field inclination angle 0° and 90°. For magnetic field inclination angle 0° (i.e. the magnetic field is parallel to the \(X\)-direction), the Lorentz force acts along the \(Y\)-direction and its magnitude is proportional to the \(Y\)-component of the velocity vector, then becomes maximum when the velocity vector is vertical, causing the elongation of the eddy. As the Hartmann number increases, the Lorentz force effect increases, consequently the elongation of the eddy becomes more significant and its axis approaches the vertical. Furthermore, the Lorentz force is opposed in direction to the \(Y\)-component of the velocity vector, which gives its retardation effect. At magnetic field inclination angle 90°, the magnetic field is vertical and the Lorentz force acts along the \(Y\)-direction and induces a horizontal elongation of the eddy. In this case, its magnitude is proportional to the \(X\)-component of the velocity vector. Also, its direction is opposed to the \(X\)-component of the velocity vector, and then reduces the strength of circulation inside the cavity.

We investigate now the case of non constant magnetic fields \((n \neq 0)\). Graphs of the evolution of the Nusselt number with the Hartmann number at the onset of natural convection are illustrated by figure 6 (the \(X\)-axis is given in logarithmic scale), for Grashof and initial Hartmann numbers equal to 10^7 and 100 respectively. Oscillations of the Nusselt number are obtained in figure 6. These fluctuations and magnitude of oscillations are important for lower values of relaxation time and diminish as the relaxation time increases \((n \text{ decreases})\).

At lower value of relaxation time, fluctuations of the Nusselt number indicate that the flow exhibits oscillatory behaviour. At the very beginning of the transient state heat transfer is mainly due to heat conduction, since the Nusselt number is equal to unity. The isotherms are nearly parallel to the active walls generating a horizontal temperature gradient.

The streamlines are those of a single spiral with its center being at the center of the cavity. As time proceeds the isotherms are gradually deformed by convection generating a vertical temperature gradient while the horizontal temperature gradient diminishes in the center of the cavity and becomes locally negative which causes an elongation of the central streamline and the development of a second spiral in the core. The transition from a single-cell to double-cell configuration may induce generation of internal waves in the temperature fields who can be at the origin of the oscillations of the whole cavity.
The current result is consistent with the findings of Ivey [27] and Schladow [28] who showed the existence of transient oscillations in enclosures consisting of two isothermal vertical walls and two adiabatic horizontal walls. Ivey [27] claimed the transient oscillations occurred because of an internal hydraulic jump with an increase of the horizontal intrusion layers. These oscillations were stated to disappear as the interior is set in motion and stratifies in temperature, increasing the thickness of the intrusion and flooding the hydraulic jump. Transient oscillations consisting of two distinct boundary layer instabilities and a whole cavity oscillation were observed by Schladow [28]. The whole cavity oscillations were attributed to the horizontal pressure gradient established by changes in the intrusion temperature field. By increasing the relaxation time (decreasing n), the magnitude and the number of oscillations of the Nusselt number decrease. This is due to the fact that the transition from a single-cell to double-cell configuration is made gradually (not sudden), so inducing a decrease of the amplitude of the generated internal thermal waves in the cavity. At critical relaxation time $\chi = 1/20$, the instability of the Nusselt number becomes insignificant and therefore the internal thermal waves disappears in the cavity. It’s important to note that for all used values of relaxation time (with the exception of the case $n = 0$) the Nusselt number reaches the same constant value. This is due to the fact that the magnetic field becomes absent (zero) in the stationary state. Also, from figure 6, one can see that the stage of the pure conduction regime increases with increasing relaxation time of the magnetic field. This can be explained that for constant dimensionless time the Hartmann number increases with increasing relaxation time inducing an increase of the Lorentz force which retards the birth of the convection regime and decreases its stage, before reaching the steady state. The evolution of stream lines with time in the transient convective regime is given by figures 7 and 8. The Grashof number, the initial Hartmann number and the inclination magnetic field angle are kept constant and equal to $10^5$, 100 and 0° respectively.

Fig. 6. Nusselt number distribution as function of logarithmic coordinate dimensionless time for different value of n at $Ha_0 = 100$, $Gr = 10^5$ and $\alpha = 0°$

Fig. 7. Stream lines for $Ha_0 = 100$; $n = 0$; $\alpha = 0°$ and $Gr = 10^5$ as function of dimensionless time.
The relaxation time of used magnetic field takes two values $1/50$ ($n = 50$) and $1/20$ ($n = 20$). It can be seen from these figures that at the very beginning of the transient regime ($\tau = 0.02$) a vertical elongated single eddy was developed in the center of the cavity. The elongation of the single eddy is more pronounced for $n = 20$ then for $n = 50$. In fact, at zero magnetic field inclination angle, the Lorentz force acts along the Y-direction and therefore causes a vertical elongation of the eddy. At the same time ($\tau = 0.02$), the vertical elongation is more noticeable for relaxation time equal to $1/20$, then for relaxation time equal to $1/50$. On the other hand the stream lines values are less noticeable for relaxation time equal to $1/20$, then for relaxation time equal to $1/50$. This is due to the corresponding higher magnitude of the magnetic field at relaxation time equal to $1/20$, which conduct to more pronounced effect of the Lorentz force and therefore leads to more significant vertical elongation of the eddy, and more effect retardation of the fluid motion. It can be concluded from figures 7 and 8 that the transition from a single-cell to double-cell configuration is made as faster as the relaxation time decreases. This is due to the fact that the effect of the Lorentz force was suppressed and consequently the stationary state was reached as quicker as the relaxation time decreases. It’s important to notice that, for relaxation time equal to $1/50$, an oscillation of the flow patterns (same contour maps at $\tau = 0.065$ and $\tau = 0.09$) which induces oscillation in velocity and thermal fields are obtained. Whereas no oscillation was observed at relaxation time equal to $1/20$, following the supple transition form a single-cell to double-cell configuration, due to the effect of the Hartmann number.

V. Conclusions

Imposed evanescent magnetic field acting on Newtonian Boussinesq incompressible fluid enclosed in heated square cavity was considered in this study. The values of relaxation time of the magnetic field are chosen, so that the magnetic field acts only in the transient state of natural convection. The validation of numerical results was presented at constant magnetic field. The results about the evanescent magnetic field can be presented as follows:

1. The fluctuations and magnitude of oscillations of the Nusselt number are important for lower values of relaxation time and diminish as the relaxation time increases.
2. Fluctuations of the Nusselt number were observed at lower value of relaxation time, indicating that the flow exhibits oscillatory behavior.
3. The magnitude and the number of oscillations of the Nusselt number decrease when the relaxation time increases.
4. A critical relaxation time value $\chi = 1/20$ is obtained for which the instability of the Nusselt number becomes insignificant.
5. The stage of the pure conduction regime increases with increasing relaxation time of the magnetic field.
6. The transition from a single-cell to double-cell configuration is made faster as the relaxation time decreases.
7. No oscillation was observed at relaxation time equal to 1/20, following the supple transition form a single-cell to double-cell configuration, due to the effect of the Lorentz force.

References

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