MAGNETIC FIELD EFFECTS ON THE FLOW PATTERNS AND THE ENTROPY GENERATION OF NATURAL CONVECTION IN SQUARE CAVITY

MOURAD MAGHERBI*, ATEF EL JERYb and AMMAR BEN BRAHIMb

*Higher Institute of Applied Sciences and Technology of Gabes
Omar Ibn El Khattab Street
6029 Gabes, Tunisia
e-mail: magherbim@yahoo.fr

bNational School of Engineers of Gabes
Omar Ibn El Khattab Street
6029 Gabes, Tunisia

Abstract

This paper investigates the effect of an imposed magnetic field on the flow patterns, and the entropy generation in a square cavity. A control volume finite element method is used to solve the conservation equations at Prandtl number of 0.71. The effects of Grashof number, Hartmann number and inclination angle of the magnetic field are investigated. The study covers the range of the Hartmann number from 0 to 50, the magnetic field inclination angle from 0° to 90° with Grashof number ranging between 10^3 and 10^5. The effects of Hartmann number and the magnetic field inclination angle are presented graphically in terms of isotherm and streamline plots. The effect of the magnetic field is found to suppress the convection currents and heat transfer inside the cavity. This effect is significant for high Grashof numbers. Results show that the Nusselt number is clearly affected by the magnetic field. The effect of the Hartmann number on entropy generation was

Keywords and phrases: entropy generation, heat transfer, convection, magnetic field, cavity, numerical method.

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INVESTIGATION OF HYDROMAGNETIC DOUBLE-DIFFUSIVE CONVECTION IN A RECTANGULAR ENCLOSURE WITH OPPOSING TEMPERATURE AND CONCENTRATION GRADIENTS

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Greek Symbols

\( \sigma \)  

electrical conductivity of the fluid

\( \alpha \)  
magnetic field inclination angle

\( \beta \)  
coefficient of thermal expansion (K\(^{-1}\))

\( \theta \)  
dimensionless temperature

\( \mu \)  
dynamic viscosity (kg \cdot m\(^{-1}\) \cdot s\(^{-1}\))

\( \nu \)  
cinematic viscosity (m\(^2\) \cdot s\(^{-1}\))

\( \varphi \)  
cavity inclination angle

\( \tau \)  
dimensionless time

\( \Omega \)  
system volume

Subscripts

\( a \)  
dimensionless

\( c \)  
cold wall

\( h \)  
heat transfer/hot

\( f \)  
friction effect

\( l \)  
local

\( p \)  
steady state

\( T \)  
total

1. Introduction

The influence of magnetic field on natural convection in square and/or rectangular enclosures has received considerable attention in the recent years, due to many important engineering and geophysical applications. The use of an external magnetic field has increasing applications, such as the processing of bulk semiconductor mono-crystals and other technological processes involving metal melting and solidification. Chamkha and Al-Naser [7] investigated the hydromagnetic double-diffusive convection in a rectangular enclosure with opposing temperature and concentration gradients. Numerical results are reported for the effect of the heat generation or absorption coefficient and the Hartmann number on the contour of streamline, temperature, concentration, and density. Rudraiah et al. [14] used modified alternating direction implicit (ADI) finite difference scheme to solve the

Nomenclature

\( a \)  
thermal diffusivity (m\(^2\) \cdot s\(^{-1}\))

\( B \)  
magnetic field (T)

\( E \)  
electrical field

\( g \)  
acceleration due to gravity (m\(^2\) \cdot s\(^{-2}\))

\( Gr \)  
thermal Grashof number

\( Ha \)  
Hartmann number (Ha = B \cdot L / \sqrt{\sigma / \mu})

\( J \)  
flux density vector

\( J \)  
current density

\( k \)  
conductivity (J \cdot m\(^{-1}\) \cdot s\(^{-1}\) \cdot K\(^{-1}\))

\( L \)  
cavity length (m)

\( p \)  
pressure (N \cdot m\(^{-2}\))

\( P \)  
dimensionless pressure

\( Pr \)  
Prandtl number

\( q \)  
heat flux

\( S \)  
entropy generation per unit volume (J \cdot m\(^{-3}\) \cdot s\(^{-1}\) \cdot K\(^{-1}\))

\( t \)  
time (s)

\( T \)  
temperature (K)

\( T_0 \)  
bulk temperature (T\(_b\) = (T\(_h\) + T\(_c\))/2)

\( \Delta T \)  
temperature difference (\( \Delta T = T_h - T_c \))

\( V \)  
velocity vector

\( \nu \)  
dimensionless velocity vector

\( u, v \)  
velocity components in x, y directions (m \cdot s\(^{-1}\))

\( U, V \)  
dimensionless velocity, components in x, y directions

\( x, y \)  
Cartesian coordinates (m)

\( X, Y \)  
dimensionless Cartesian coordinates

investigated in steady-unsteady state of natural convection. It was found that entropy generation in the steady state decreases when Hartmann number increases. Additionally, the oscillatory behavior of the entropy generation at the onset of natural convection can be suppressed at critical Hartmann number.
vorticity-stream function formulation of natural convection inside a rectangular enclosure in the presence of a magnetic field. The numerical results are obtained for a large range of Grashof and Hartmann numbers with Prandtl number \( Pr = 0.733 \). Dominant convection with is predicted for high Grashof and low Hartmann numbers. Results show that the magnetic field suppresses the rate of convective heat. Alchaar et al. [3] numerically studied two-dimensional natural convection in a shallow cavity heated from below with the presence of inclined magnetic field. They showed that the magnetic field reduces the heat transfer and inhibits the onset of the convection current. Furthermore, the convection mode inside the cavity is found to depend strongly upon both the strength and orientation of the magnetic field. Al-Najem et al. [4] used the power low control volume approach to determine the flow and temperature fields under a transverse magnetic field in a tilted square enclosure with two isothermal and two adiabatic walls. The magnetic field effects on convection currents and heat transfer is more significant for low inclination angles and high Grashof numbers. Tagawa et al. [17] numerically solved the magnetizing force for natural convection of air in a cubic enclosure. Results show that the convection of fluid can be explained by the repulsion of hot fluid from the hot wall to weak magnetic field and the attraction of cold fluid to the strong magnetic field. Hossain et al. [10] numerically investigated the effect of surface tension on unsteady natural convection flow of an electrically conducting fluid in a rectangular enclosure under an externally imposed magnetic field with internal heat generation. They showed that a change of direction of the external magnetic field force from horizontal to vertical leads to decrease the flow rates and causes an increase in the effect of the thermo-capillary force. Cem Ece and Büyük [6] studied a laminar natural convection flow in the presence of a magnetic field in an inclined rectangular enclosure heated from left vertical wall and cooled from the top wall, while the other walls are kept adiabatic. Results showed that the flow characteristics and therefore convection heat transfer inside the tilted enclosure depend strongly upon the strength and direction of magnetic field, the aspect ratio and the inclination of the enclosure. The circulation and the convection are significantly suppressed by the presence of a strong magnetic field. Nevertheless, investigating entropy generation in convective heat transfer has received growing attention in the last few years. Bejan [5] showed that the entropy generation for forced convective heat transfer is due to temperature gradient and viscous effect in the fluid. Sahim [16] analytically investigated entropy generation in turbulent liquid flow through a smooth duct subjected to constant wall temperature. It was found that constant viscosity assumption may yield a considerable amount of deviation on entropy generation. Magherbi et al. [12] investigated the entropy generation at the onset of natural convection. They showed that the total entropy generation has a maximum value at the onset of the transient state, which increases with the Rayleigh number and the irreversibility distribution ratio. It was found that entropy generation asymptotically tends towards a constant value at low Rayleigh numbers, whereas an oscillation of the entropy generation was observed for higher Rayleigh numbers, before reaching the steady state. It was found that beyond a critical value of the Rayleigh number, the Prigogine's theorem of minimum entropy production is not verified and the system is out of the linear branch of irreversible phenomena. Abbassi et al. [1] investigated the entropy generation in Poiseuille-Benard channel flow. Results show that entropy generation is largely higher near the channel walls than that in the central flow and its maximum is located just near the walls in the regions where Nusselt number is maximum.

The present paper reports a numerical study about the impact of an imposed magnetic field on the flow patterns at steady state of natural convection and its magnitude influence on entropy generation at the onset of natural convection, which is not yet been encountered. The governing flow and energy equations are considered for two-dimensional case. A numerical method is conducted based on the Control Volume Finite-Element. The resulting streamlines distributions are investigated as a function of three independent parameters: inclination angle of the magnetic field, Grashof and Hartmann numbers. The evolution of entropy generation in transient state is investigated for Hartmann number ranging from 0 to 100, at zero magnetic field inclination angle, for irreversibility distribution ratio equal to \( 10^{-5} \) and for Grashof number equal to \( 10^5 \). The entropy generation is also calculated in steady state, for various values of Hartmann number.
2. Mathematical Formulation

Imposed magnetic field on laminar natural convective fluid enclosed in a square cavity is considered in this problem. Figure 1 shows the domain to be analyzed in adopted coordinate system. Upper and lower walls, parallel to the horizontal axis, are adiabatic, while the left and the right walls are isothermal. This fluid is modeled as a Newtonian and Boussinesq incompressible fluid, whose properties are described by its kinematics viscosity \( \nu \), thermal diffusivity \( \alpha \) and thermal volumetric expansion coefficient \( \beta_T \). We further assume that the cavity is submitted to a uniform inclined magnetic field \( B \). Its inclination angle with the horizontal axis is \( \alpha \).

The electric current \( J \) induced by the magnetic field is defined by:
\[
J = \sigma (E + \mathbf{V} \times B) \tag{1}
\]
The detailed derivation of the above equation is available in Woods [18]. It is assumed that the electric force per unit charge (\( E \)) is negligible compared to the magnetic force per unit charge (\( \mathbf{V} \times B \)).

Under the above assumptions, the conservation equations for mass, momentum and energy in a two-non-dimensional form are as follows:
\[
\text{div}(\mathbf{u}) = \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{2}
\]
\[
\frac{\partial U}{\partial t} + \text{div}(\mathbf{J}_U) = -\frac{\partial P}{\partial X} + \left[ \text{Gr}_T \cdot \theta \right] \cos \varphi + \text{Ha}^2 (U \cdot \sin \alpha \cdot \cos \alpha - U \cdot \sin^2 \alpha), \tag{3}
\]
\[
\frac{\partial V}{\partial t} + \text{div}(\mathbf{J}_V) = -\frac{\partial P}{\partial Y} + \left[ \text{Gr}_T \cdot \theta \right] \sin \varphi + \text{Ha}^2 (U \cdot \cos \alpha \cdot \sin \alpha - V \cdot \cos^2 \alpha), \tag{4}
\]
\[
\frac{\partial \theta}{\partial t} + \text{div}(\mathbf{J}_\theta) = 0, \tag{5}
\]
where
\[
\mathbf{J}_U = U \mathbf{u} - \nabla \cdot \mathbf{U}, \tag{6}
\]
\[
\mathbf{J}_V = V \mathbf{u} - \nabla \cdot \mathbf{V}, \tag{7}
\]
\[
\mathbf{J}_\theta = \theta \mathbf{u} - \frac{1}{Pr} \nabla \theta, \tag{8}
\]
where the dimensionless variables are defined by
\[
X = \frac{x}{L}; \quad Y = \frac{y}{L}; \quad U = \frac{uL}{a}; \quad V = \frac{vL}{a}; \quad \theta = \frac{T - T_0}{T_h - T_c}; \quad P = \frac{pL^2}{\rho a^2}; \tag{9}
\]
\[
\text{Gr}_T = \frac{g^2 \beta_T \Delta T L^3}{\nu^2}; \quad \tau = \frac{aL}{L^2}; \quad \text{Ha}^2 = \frac{B^2 L^2 \sigma}{\mu}. \tag{9}
\]
The valued average Nusselt number like as follows:
\[
\text{Nu} = \int_0^1 \left( \frac{\partial \theta}{\partial Y} \right) dx. \tag{10}
\]

The boundary conditions appropriate to laminar flow within the differential heated cavity are:
\[
U = V = 0 \quad \text{for all walls},
\]
\[
\theta = 0.5 \quad \text{on plane } X = 0,
\]
\[
\theta = -0.5 \quad \text{on plane } X = L,
\]
\[
\frac{\partial \theta}{\partial Y} = 0 \quad \text{on planes } Y = 1 \text{ and } Y = 0.
\]
The initial conditions are:
\[
\text{At } \tau = 0, \quad U = V = 0, \quad P = 0 \quad \text{and } \theta = 0.5 - X \text{ for whole space}.
\]

3. Numerical Procedure

A modified version of the Control Volume Finite-Element Method (CVFEM) of Saabas and Baliga [15] is adapted to the standard staggered grid in which pressure and velocity components are stored at different points. The SIMPLER algorithm was applied to resolve the pressure-velocity coupling in conjunction with an Alternating Direction Implicit (ADI) scheme for performing the time evolution. From the known temperature and velocity fields at any instant \( \tau \) given by solving equations (2)-(5), the dimensionless average Nusselt number for the entire cavity \( \text{Nu} \) is easily obtained by equation (7). The shape function describing the variation of the dependant variable \( \psi \) (= \( U, V, \theta \)) is needed to calculate the flux across the control-volume faces. We have
followed Saabas and Baliga [15] in assuming linear and exponential variations, respectively, when the dependant variable $v$ is calculated in the convective term of the conservation equations. More details and discussions about CVFEM are available in the works of Prakash [13], Hooke [9], Elkaim et al. [8], Saabas and Baliga [15] and in many other works. The numerical code used here is described and validated in details in Abbassi et al. [1].

4. Results and Discussion

The present study is restricted to the non-reactive and Newtonian fluids with Prandtl number equal to 0.71. The induced magnetic field is neglected. No electric field is assumed to exist and the Hall effect is negligible. For the validation of our numerical results, streamlines and isotherms in square enclosure are plotted in Figure 2; also the average Nusselt number is compared with some previous numerical results in Table 1, in absence of magnetic field. It can be seen that for smaller Grashof number ($10^3$ and $10^4$) one cell was developed in the centre of the cavity, whereas by increasing the Grashof number ($10^5$) two cells appears in the cavity: one at the top of the hot wall and another at the bottom of the cold wall. The deformation of isothermal lines which are more pronounced for Grashof number equal to $10^5$ appears due the high convective current inside the cavity. It can be seen from Figure 2 and Table 1 that our results are in good agreement with the work of Al-Najem et al. [4].

4.1. Effects of varying the Grashof and Hartmann numbers on the flow patterns

We investigate first the effect of a magnetic field on the flow patterns of an electrically conducting fluid in square cavity. The Grashof and Hartmann numbers are ranging, respectively, from $10^3$ to $10^5$ and from 0 to 50. Typical value of direction of the magnetic field with the horizontal considered to be (0°, 30°, 40°, 60° and 90°). Streamlines for various Grashof and Hartmann numbers are given in Figures 3, 4 and 5. It can be seen from these figures that for constant value of Grashof and Hartmann numbers, the strength of circulation inside the cavity reduces when the inclination angle of the magnetic field increases from 0° to 90°. This is due to the retarding effect of the Lorentz force induced by the magnetic field. It can be found from Figures 3, 4 and 5, that for constant value of Grashof and small value of the Hartmann number ($Ha = 10$), the flow patterns seem to be similar when the magnetic field angle inclination increases from 0° to 90°. Whereas for high value of Hartmann number the flow patterns change noticeably, and one can observe a clockwise rotation of the eddy axis from the vertical to the horizontal, when increasing the inclination angle of the magnetic field from 0° to 90°. Figures 3-5 show the considerable effect of the magnetic field (Ha) on the flow patterns of the fluid. It can be seen from Figure 2, for Grashof number equal to $10^4$ at zero Hartmann number, that the flow is characterized by a single eddy circulating around the entire enclosure. As can be seen in Figure 4, for constant values of Grashof number (Gr : $10^4$) and inclination angle, the application of the magnetic field tends to slow down the movement of the fluid. Also, the single eddy is observed to be elongates and its axis to be rotates in counterclockwise (except for 90° where the rotation is clockwise). The elongation of the eddy and the rotation of its axis are seen to be more significant as the Hartmann number increases. In fact, the magnetic field applied in the XY-plane, induces an electric current in the Z-direction, which generates a magnetic force (Lorentz force) in the XY-plane. The direction and the magnitude of the Lorentz force are at the origin of the elongation and the axis rotation of the central eddy. Let us consider the two particular cases of the magnetic field inclination angle 0° and 90°. At magnetic field inclination angle 0° (i.e., the magnetic field is parallel to the X-direction) (Figure 4), the Lorentz force acts along the Y-direction and its magnitude is proportional to the Y-component of the velocity vector and becomes maximal when the velocity vector is vertical, causing simultaneously the elongation of the eddy and the rotation of its axis. As the Hartmann number increases, the Lorentz force effect increases, consequently, the elongation of the eddy becomes more significant and its axis approaches the vertical. Furthermore, the Lorentz force is opposed in direction to the Y-component of the velocity vector,
which gives its retardation effect. At magnetic field inclination angle 90° (Figure 4), the magnetic field is vertical. The Lorentz force acts along the Y-direction and induces a horizontal elongation of the eddy. In this case, its magnitude is proportional to the X-component of the velocity vector. Also, its direction is opposed to the X-component of the velocity vector, and then reduces the strength of circulation inside the cavity. Similar observations can be established, for Grashof number equal to 10^5, as can be seen in Figure 5. In this case, it is important to note that the two eddies observed for Grashof number equal to 10^5 at zero Hartmann number and zero inclination angle (Figure 2), combine together to form one central and slower recirculating eddy when increasing the Hartmann number (Ha ≥ 10). This indicates that the viscous boundary layers formed at high Grashof number and zero Hartmann number (Figure 2), become to disappear when increasing the magnetic field (Figure 5).

4.2. Hartmann number effect on the entropy generation

The existence of a thermal gradient between the vertical walls of the enclosure sets the fluid in a non-equilibrium condition which generates entropy in the system. According to local thermodynamic equilibrium with linear transport theory, the local entropy generation is given by Mahmud and Fraser [12], as:

\[ \dot{S}_l = \frac{k}{T_0} \left( \nabla T \right)^2 + \frac{\nabla \cdot \mathbf{V}}{T_0} - \frac{1}{T_0} (\mathbf{J} - Q \mathbf{V})(E + \mathbf{V} \otimes \mathbf{B}). \quad (11) \]

It is assumed that in the effective current density term (\(\mathbf{J} - Q \mathbf{V}\)) of equation (11), \(\mathbf{J} >> Q \mathbf{V}\).

Once the magnetic field is in the XY-plane and in the case of two dimensional Cartesian system equation (11) can be written as:

\[ \dot{S}_l = \frac{k}{T_0} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu}{T_0} \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \]

\[ + \frac{\sigma B^2}{T_0} (u \sin \alpha - v \cos \alpha)^2. \quad (12) \]

The local entropy generation can be made dimensionless by using the dimensionless variables listed in equation (9):

\[ \dot{S}_{l,a} = \dot{S}_{l,a,h} + \dot{S}_{l,a,f} + \dot{S}_{l,a,m}, \quad (13) \]

where

\[ \dot{S}_{l,a,h} = \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2, \quad (14) \]

\[ \dot{S}_{l,a,f} = \phi \left[ 2 \left( \frac{\partial U}{\partial x} \right)^2 + 2 \left( \frac{\partial V}{\partial y} \right)^2 + \left( \frac{\partial U}{\partial y} + \frac{\partial V}{\partial x} \right)^2 \right], \quad (15) \]

\[ \dot{S}_{l,a,m} = Ha^2 \phi (U \sin \alpha - V \cos \alpha)^2. \quad (16) \]

The irreversibility distribution ratio \(\phi\) is given by

\[ \phi = \frac{\mu T_0}{k \left( \frac{a}{L(\Delta T)} \right)^2}. \quad (17) \]

The first term on the right-hand side of equation (11) shows the local entropy generation due to heat transfer (\(\dot{S}_{l,a,h}\)), the second term shows the local entropy generation due to fluid friction (\(\dot{S}_{l,a,f}\)), while the third term is due to the magnetic field (\(\dot{S}_{l,a,m}\)). The dimensionless total entropy generation is the integral over the system volume of the dimensionless local entropy generation:

\[ \dot{S}_{T,a} = \int \dot{S}_{l,a} d\mathbf{V}. \quad (18) \]

In this section, the Grashof, Prandtl numbers, the irreversibility distribution ratio and the inclination angle of the magnetic field are fixed, respectively, at \(10^5\), 0.71, \(10^{-5}\) and 0°. The Hartmann number is ranging from 0 to 100. Numerical results for the total entropy generation in steady and unsteady states as well as selected thermal contours for various values of Hartmann number will be reported. Figure 6 shows the evolution of entropy generation in the transient state of natural convection. As can be seen from this figure, the entropy generation fluctuates before reaching the steady state, at zero Hartmann number. As
explained by Magherbi et al. [11], the oscillatory behavior of entropy generation is due to the transition from a single to a double configuration of the flow patterns at the very beginning of natural convection, which induces generation of internal waves in the velocity and temperature fields. Consequently, the system is in the case of a spiral approach towards the steady state, and evolves in the non-linear branch of irreversible phenomena. Figure 6 shows that, the amplitude and the number of entropy generation oscillations are decreased, as the Hartman number increases. Beyond a critical Hartmann number (Ha_c ~ 20), no oscillatory behavior exists. The entropy generation at the onset of the transient state tends asymptotically to a constant value in the steady state. From a thermodynamics view point, the asymptotic behavior of the total entropy generation (i.e., the system returns directly towards the steady state) with time at critical Hartmann number shows that the system is in the linear branch of the thermodynamics of irreversible processes, since the Prigogine’s theorem of minimum entropy production is verified. As a result, the Lorentz force effect seems to make the steady state sufficiently close the equilibrium state. Figure 7 depicts the steady state entropy generation with effect of increasing value of Hartmann number. It may be seen, from this figure that the total entropy generation decreases as the Hartmann number increases. This is expected since application of magnetic field retards the velocity field and thus decreases the magnetic and viscous entropy generation. The corresponding effect on the increasing Hartmann number on thermal entropy generation may be viewed by the plot of the isotherms in Figure 8. It can be seen that as Hartmann number increases, the temperature stratification in the core diminishes and the thermal boundary layers at the active walls disappear. Furthermore, the isotherms become more straighten out at high Hartmann number, and subsequently present small values of the thermal gradients, which induce insignificant entropy generation.

5. Conclusion

The effect of the magnetic field on the flow patterns and the entropy generation for natural convection was investigated numerically by using a Control Volume Finite-Element Method. The influence of the inclination angle of the magnetic field on the streamlines was examined. Results show that the convection heat transfer inside the cavity, depends strongly on the magnitude and direction of the imposed magnetic field. It was found that the Lorentz force tends to reduce the strength of circulation inside the cavity. It was established that for constant value of Grashof and small value of the Hartmann number, the flow patterns seem to be similar when the magnetic field angle inclination increases from 0° to 90°, while for relatively high value of Hartmann number, the flow patterns change noticeably. The influence of the Hartmann number on entropy generation at unsteady state was studied. Results show that oscillatory behavior of entropy generation for high Grashof numbers at the onset of natural convection begins to disappear when increasing the Hartmann number. Therefore, no oscillations are observed for a critical Hartmann number closer to 20, the Prigogine’s theorem of minimum entropy production is verified and the system evolves in the linear branch of irreversible phenomena. Results show that the entropy generation at steady state decreases when the Hartmann number increases. The decrease of entropy generation in steady state is insignificant for small value of the Hartmann number, whereas it becomes more pronounced when the Hartmann number increases.

References

Table 1. Average Nusselt number of some previous numerical results and present study

<table>
<thead>
<tr>
<th>Gr</th>
<th>Ha</th>
<th>Nu ( (\alpha = 0^\circ) )</th>
<th>References</th>
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<td>(10^4)</td>
<td>0</td>
<td>2.055</td>
<td>Tagawa et al. [17]</td>
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<td>0</td>
<td>4.339</td>
<td>Al-Najem et al. [4]</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>1.725</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>25</td>
<td>1.185</td>
<td></td>
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<tr>
<td>50 and 100</td>
<td>0</td>
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<tr>
<td>10^4</td>
<td>25</td>
<td>1.190</td>
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<tr>
<td></td>
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Figure 1. Schematic view of 2D square cavity with the imposed magnetic field.

Figure 2. Streamlines and isothermal lines for $Gr = 10^3$.
(a) Streamlines; (b) Isotherms.

Figure 3. Stream lines for $Ha$ ($10 \leq Ha \leq 50$) and $\alpha$ ($0 \leq \alpha \leq 90^\circ$) at $Gr = 10^3$. 

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Figure 4. Stream lines for $\text{Ha} (10 \leq \text{Ha} \leq 50)$ and $\alpha (0 \leq \alpha \leq 90^\circ)$ at $\text{Gr} = 10^4$.

Figure 5. Stream lines for $\text{Ha} (10 \leq \text{Ha} \leq 50)$ and $\alpha (0 \leq \alpha \leq 90^\circ)$ at $\text{Gr} = 10^5$.

Figure 6. Evolution of the total entropy generation with time for $\text{Gr} = 10^5$, $\phi = 10^{-5}$ and $\alpha = 0$. 
Figure 7. Entropy generation in steady state versus Hartmann number for $\alpha = 0^\circ$, $\varphi = 10^{-6}$ and $Gr = 10^5$.

Figure 8. Isothermal lines for $Ha (0 \leq Ha \leq 100)$, $\alpha = 0^\circ$ and $Gr = 10^5$. 