This paper investigates the dynamic stability of a power system distribution network integrating variable speed wind generation based on a bifurcation analysis. A generalized model of the system is developed taking into account multiple wind farms, transformer branches and composite dynamic load model. Bifurcations may be analyzed by a systematic testing of the complete system Jacobian matrix. Considering the load voltage, frequency and voltage gradient dependency, three models are studied. Factors of impact are: the generator input mechanical power, the loading factor and the reactive loading. It is shown that generator and load dynamic interaction may trigger nonlinear phenomena, such as voltage oscillations and the appearance of bifurcations.

Index Terms- Bifurcation, doubly fed induction generator, static load, dynamic load, distribution network.

1. INTRODUCTION

The growing integration of wind energy into power system distribution networks is sought to provide for the system operator a distributed and reduced loss power supply which retain the system voltage and dynamic stability. For these related reasons among which its reactive power control capabilities [1], the development of wind turbine techniques, has led to an extensive use of the doubly fed induction generator (DFIG) in wind farms [2-14].

However, distribution networks are generally characterised by a variable load composition ranging from quasi static residential, commercial loads, to large motor industrial loading. Reactive power demand generally increases with load increase, motor stalling or a change in load composition, which depend on the load nodal voltage, voltage gradient and frequency. The interaction between the induction generator dynamics and static or dynamic loads in the distribution networks may trigger nonlinear phenomena, such as voltage oscillations and the appearance of bifurcations [3-5].

Many research studies have investigated the stability of distribution networks integrating DFIG wind farms. The paper of [3] considered a static analysis based on continuation power flow for identifying the maximum load increment before reaching voltage instability in the distribution network including a significant number of distributed energy resources. The work in [4] presents an eigenvalue analysis when the wind farm penetration is increased and a voltage regulation loop is considered.

The report of [5] deals with the impact of doubly fed induction generators on the stability of a power system. Numerical simulations reveal that the addition of doubly fed generators, such as those in wind parks to a power system improves the response of the system after small disturbances, but can worsen it after larger disturbances. In [6], the impact of the distributed generators on the oscillatory stability is studied to determine the operating stability limits under small and continual predictable changes in loading conditions.

The thesis of [7] analyzed small disturbances which force wind turbine generators to exhibit oscillatory instability and concluded that the DFIG has no significant impact on oscillation, whereas constant speed wind turbine generator has an affect on oscillation and adds damping to the system. The main purpose of [8] is to investigate the influence of long distance transmission line on the eigenproperties of a DFIG.

In most of these studies the load has been considered as constant PQ load; the dynamics are neglected. This assumption may be adequate for distribution networks feeding mainly residential and commercial loads. However, wind farms are installed in the vicinity of large dynamic load consumption points such as irrigation and industrial plants.

Nonlinear dynamic phenomena of such systems may be better understood by a generalized approach using the standard theory of bifurcation. The purpose of this paper is to presents a bifurcation analysis of power system distribution networks taking into account distributed induction generator and the load frequency, voltage gradient and angle dynamics to investigate the stability of a distribution network.

The distribution network incorporating the variable speed wind generator is represented by three different models: a differential-algebraic model (DAE) with an infinite bus, a DAE model with a voltage dependent static load and a DAE model with a fully dynamic load. Section 2 presents the doubly fed induction generator, network and load models. In section 3, we develop a generalized dynamic model of the DFIG and its control system in a distribution network. In section 4, we identify the different bifurcation and stability problem in a differential-algebraic power system model. In section 5, various simulation results are presented where the bifurcation parameters are: the induction generator input mechanical power, the static loading factor, and the reactive power loading are varied. Conclusions and future work are given in section 6.
2. DISTRIBUTION NETWORK MODELLING

2.1 Doubly fed induction generator and control model

The DFIG is represented by a third-order model where the state variables used are: the rotor slip $s$, the magnitude and the angle of the internal generator voltage $E$ and $\delta$ respectively. Inertial dynamics are expressed using a single-mass model of the turbine and generator rotor system.

The dynamic model written in matrix form may be represented with the differential system [5-6]:

$$\dot{\delta} = \frac{1}{E_s} \left[ \frac{1}{X_m} + \frac{1}{X_s} \right] \left( \frac{1}{X_m} \frac{1}{X_s} \right) \left( V \sin(\delta - \alpha) + \frac{1}{2H} \frac{V_m}{V} \left( E - \frac{V_m}{V} \right) \right)$$

(1)

$$\dot{E} = \frac{1}{E_s} \left[ \frac{1}{X_m} + \frac{1}{X_s} \right] \left( \frac{1}{X_m} \frac{1}{X_s} \right) \left( V \sin(\delta - \alpha) + \frac{1}{2H} \frac{V_m}{V} \left( E - \frac{V_m}{V} \right) \right)$$

Where $E = E_s^2 X_r / X_r'$ and $P_r = E_m / Y_r$.

In the above system, $\omega_s$ is the synchronous angular velocity, $E$, $\delta$ and $s$ are respectively the magnitude, the angle of the transient voltage and the slip of the DFIG. $Y$, $V_r$, $\alpha$ and $\theta_r$ are respectively the magnitude and the angle of the terminal and rotor voltage. $X_s$, $X_r$ are respectively the stator and rotor leakage reactance. $X_m$ is the magnetizing reactance, $X_s'$ is the induction generator transient reactance. $T_0$ is the induction generator open circuit time constant. $P_m$ is the input mechanical power and $H$ is the inertia constant in seconds. Here, $P_r$ is constant throughout simulations and $F$ is determined by the power flow in the system.

Converter dynamics are highly simplified, as they are fast with respect to the electromechanical transients [3]. Thus, the converter is modelled as an ideal current source, where $i_{rq}$ and $i_{rd}$ are the state variables and are used for the rotor speed control and the voltage control, respectively. Differential equations for the converter currents are as follows [3]:

$$\dot{i}_{rq} = \left( -\frac{x_s}{x_m} + \frac{p_{ro}^o}{x_m^2} \right) \left( \frac{1}{\tau_i} \right)$$

(2)

$$\dot{i}_{rd} = K_v (V - V_{ref}) - \frac{V}{x_m} \dot{i}_{rd}$$

(3)

Where $K_v$ is the gain of the voltage control loop; $V_{ref}$ is the reference voltage; $\tau_i$ is the power control time constant and $p_{ro}^o$ is the power speed characteristic.

2.2 Network model

The equivalent Π-circuit representation of a general branch is shown in Fig. 1 which can be a transmission line, tap changing transformer, or phase-shifter represented by the complex value $\gamma_{ij}$ [13].

\[ V_{ij} < \theta_i \]
\[ V_k = V_i < \alpha_{ij} \]
\[ Y_{ij} = G_{ij} + jB_{ij} \]
\[ V_{Cij} \]
\[ P_j + jQ_j \]
\[ P_i + jQ_i \]
\[ P_i + jQ_i \]
\[ P_j + jQ_j \]

Figure 1. Equivalent Π circuit of a general Branch between two buses.

2.3 Dynamic load model

The load bus, with voltage magnitude $V_L$ and phase $\delta_L$, consists of an induction motor, with a constant PQ load in parallel [9]. The equation which represents the variation of the load bus angle and the voltage is given respectively by:

$$K_{qv} \delta_L = -K_{qv} V_L^2 - K_{qL} V_L \cos(\delta_L)$$

$$+ E X_v L \cos(\delta_L - \delta) - \left( Y_0 + X_s' \right) V_L^2 - (Q_0 + Q_L)$$

(4)

$$TK_{qv} P_0 V_L = K_{pv} Q_0 V_L^2$$

$$+ V_L \left( K_{pv} K_{qv} - K_{qv} K_{pv} \right)$$

$$+ \sqrt{K_{qv}^2 + K_{pv}^2} \left[ -E_0 Y_0 V_L \cos(\delta_L - \eta) \right.$$

$$+ \left. (Y_0 + X_s') V_L \cos(\delta_L - \delta - \eta) \right) + (Y_0 + X_s') V_L \cos(\eta) V_L^2$$

$$- K_{qv} \left( P_0 + P_L \right) + K_{pv} Q_0 + Q_L$$

Where $T$, $K_{pv}$, $K_{qv}$, $K_{qv}$, and $K_{qv}$ are constants of the motor, $P_0$, $Q_0$ and $P_L$, $Q_L$ are the static active and reactive power drained by the motor and by the load P-Q, respectively.

3. GENERALIZE MODEL

Figure 2 shows the general diagram of a DFIG and its control system in a distribution network. The control systems include (a) rotor speed regulator and (b) voltage control [3].
The dynamics of a power system are characterized by two types of equations; the differential equations of the dynamics of the DFIGs [5-6], and control systems. The algebraic equations represent the active and reactive power balance at the internal and terminal buses of machines, at the transformer high-side buses of DFIG, and at the load buses. Hence, the specific power system dynamic model [9] and its Jacobian matrix at an equilibrium point can be represented as follows:

\[
X = f(X, Y, \mu)
\]

\[
\theta = g(X, Y, \mu)
\]

\[
J = \begin{bmatrix} F_x & F_y \\ G_x & G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial X} & \frac{\partial f}{\partial Y} \\ \frac{\partial g}{\partial X} & \frac{\partial g}{\partial Y} \end{bmatrix} \begin{bmatrix} x_e^* & y_e^* & \mu_e^* \end{bmatrix}
\]

(7.a)

Where \(X \in \mathbb{R}^n\) is the vector of dynamic states of generators and control systems; \(Y \in \mathbb{R}^m\) is the vector of algebraic variables, i.e. the voltage and angle at each terminal bus \((V_T, V_H)\), high-side bus \((V_H, V_H)\), and load bus \((V_L, V_H)\); \(\mu \in \mathbb{R}^l\) is the vector of system parameters which vary slowly and continuously, such as wind speed \((V_w)\), and load demand parameters \((P_{DQ})\).

The Jacobian matrix of (6) at any equilibrium point \((x_e, y_e, \mu_e)\) can be obtained from the linearized power system dynamic model, which can be structurally represented as the following matrix form:

\[
\begin{bmatrix} \Delta X_{DFIG} \\ \Delta X_y \\ \Delta x_e \\ -\Delta \rho_C \\ -\Delta \rho_Q \end{bmatrix} = \begin{bmatrix} A_{DFIG} & 0 & A_{X0} & A_{XV} \\ 0 & A_N & 0 & 0 \\ A_{DFIG} & 0 & 0 & A_N \\ 0 & 0 & I_{P0} & I_{PV} \\ 0 & 0 & I_{Q0} & I_{QV} \end{bmatrix} \begin{bmatrix} \Delta X_{DFIG} \\ \Delta X_y \\ \Delta x_e \\ -\Delta \rho_C \\ -\Delta \rho_Q \end{bmatrix}
\]

(7.b)

Where, \(\Delta X_{DFIG}\) are the states of the magnitude and the angle of the transient voltage and the slip of the DFIG. \(\Delta x_e\) and \(\Delta x_q\) are respectively the states of the control system.

\[
\Delta \theta = \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \Delta \theta_3 \\ \Delta \theta_4 \end{bmatrix} \quad \text{and} \quad \Delta V = \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}
\]

are respectively the angle and voltage variables at network buses.

\[
\Delta R_C = \begin{bmatrix} \Delta R_{C1} \\ \Delta R_{C2} \\ \Delta R_{C3} \end{bmatrix} \quad \text{and} \quad \Delta Q_C = \begin{bmatrix} \Delta Q_{C1} \\ \Delta Q_{C2} \end{bmatrix}
\]

are respectively the coefficients of non-voltage-dependent reactive and active power load demand model.

The constants value \(k_i\) are expressed in the Appendix.

4. Bifurcations in Power System Dynamic Model

The following flow chart (Figure 3) presents the five types of bifurcation in the power dynamic model. In this paper, bifurcation points are identified through the eigenvalue loci of the equivalent system Jacobian matrix \(J_e\). When the determinant of \(J_e\) is zero, the equilibrium point \((x_e^*, y_e^*, \mu_e^*)\) becomes a bifurcation point and voltage collapse will occur. These bifurcations are the saddle-node bifurcation and the Hopf bifurcation. The definitions of these bifurcations are given in [10].

5. CASE APPLICATION

The effect of the induction generator input mechanical power, the static loading factor and the reactive power loading is evaluated through three models (figure 4-6): Model 1 represents the DFIG fed by a constant voltage source through a transmission line represented by a reactance [12], whereas the second and the third model consists of the DFIG connected respectively to a static and dynamic load. Time responses of the DFIG speed and the internal voltage are plotted to validate the system stability as observed in the eigenvalues analysis.

5.1. Model 1: Two-bus system

Figure 7 shows the variation of the state variable of the system in the base case. The simulation results show that the solutions of the power system model converge to the equilibrium characterized by the nominal slip equal to \(s = -1.113\%\). Figure 8 depicts the changes of relevant eigenvalues of the reduced Jacobian matrix \(J_x\) as the induction generator mechanical torque increases.
Where: s: singular; RD: row dependent; EV eigenvalue.

\[ J_y = G_y - G_x F_y^T F_y \] the equivalent static/algebraic Jacobian matrix. 
\[ J_x = F_x - F_y G_y^T G_x \] the equivalent system Jacobian matrix.

A complex conjugate pair of eigenvalue crosses the imaginary axis for \( P_m = 1.98 \) p.u with an oscillation frequency of 0.9119 Hz and negative damping ratio of -0.4453, thus leading to Hopf bifurcation (Table 1). The system does not present a stable equilibrium point and shows an oscillatory behaviour as expected from the figure 9. The oscillation continues to grow, leading to instability.

5.2. Model 2: Effect of the loading factor

Figure 10 depicts the changes of relevant eigenvalues of \( J_x \) as the loading factor for the large industrial motor increases. Note that, the static load model can be described by the following polynomial equations [11]:

\[ P_i = P_{in} \frac{V_i^a}{V_o} (1 + \lambda), \quad Q_i = Q_{in} \frac{V_i^b}{V_o} (1 + \lambda) \] (9)

Where \( P_{in}, Q_{in} \) are the values at rated voltage \( V_o, a, b \) are respectively the active and reactive power voltage exponents, and \( \lambda \) is the loading factor. One eigenvalue crosses the imaginary axis for \( \lambda = 4.0 \), thus leading to a saddle node bifurcation. Observe that, at the SN bifurcation point, the other two eigenvalues of the system are negative, thus the other system dynamics are stable. Figure 11 depicts the time domain simulation for a change of loading factor. The system undergoes a voltage collapse induced by the saddle node bifurcation. The internal voltage collapse is caused by the large industrial static load. Also the DFIG angle presents an unstable trajectory.

5.3. Effect of dynamic load models

The load reactive power is chosen as the system parameter, so that increasing \( Q_1 \) corresponds to increasing the load reactive power demand to examine its effect on the system response. The load parameter values are given in [9]. Figure 12 depicts simulation times for model 3. Results show that the solutions of the power system model converge to the equilibrium. Figure 13 depicts the changes of relevant eigenvalues of the reduced Jacobian matrix \( J_x \) as the load reactive power demand \( Q_1 \) of the dynamic load increases. One eigenvalue crosses the imaginary axis for \( Q_1 = 4.18822 \) p.u, thus leading to a saddle node bifurcation. Figure 14 depicts the time domain simulation for a change of \( Q_1 \). For \( t > 0.05s \) the system undergoes a voltage collapse induced by the saddle node bifurcation. The voltage collapse is caused by the load dynamic. However, observe that as a consequence of the voltage instability, also the dynamic load angle presents an unstable trajectory.

6. CONCLUSION

In this paper, we have investigated the dynamic stability of a power system distribution network integrating variable speed wind generation based on a bifurcation analysis. The distribution network incorporating a doubly fed induction generator was represented by three different models: a differential algebraic model with an infinite bus, a differential-algebraic model with a voltage dependent static load, and with a fully dynamic load. Various simulation results were presented for the bifurcation parameters: the induction generator input mechanical power, the static loading factor and the reactive power loading. Time domain analysis has validated the system stability as observed in the frequency domain analysis.

As a continuation to this work, we intend to extend this study to investigate the effect of a wind park including various types of variable speed wind generators as synchronous generators and doubly fed induction generator on power system stability.
Figure 7. Time response of $E, \delta$ and $s$ in the base case.

Figure 8. Eigenvalues locus for model 1.

Figure 9. System oscillations for the 3-bus system due to Hopf Bifurcation.

TABLE 1. System Modes when $P_m$ is increased.

<table>
<thead>
<tr>
<th>Eigenvalues $\lambda$</th>
<th>Frequency $f$ (Hz)</th>
<th>Damping ratio $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7.81±9.88</td>
<td>1.5724</td>
<td>0.62013</td>
</tr>
<tr>
<td>-7.53±9.76</td>
<td>1.5533</td>
<td>0.6108</td>
</tr>
<tr>
<td>-7.46±10.15</td>
<td>1.6154</td>
<td>0.5922</td>
</tr>
<tr>
<td>-7.22±11.08</td>
<td>1.7634</td>
<td>0.5459</td>
</tr>
<tr>
<td>-6.25±11.96</td>
<td>1.9034</td>
<td>0.4631</td>
</tr>
<tr>
<td>-5.47±12.11</td>
<td>1.9273</td>
<td>0.4116</td>
</tr>
<tr>
<td>-3.91±11.76</td>
<td>1.8716</td>
<td>0.3155</td>
</tr>
<tr>
<td>-2.21±10.80</td>
<td>1.7188</td>
<td>0.2004</td>
</tr>
<tr>
<td>-0.11±9.12</td>
<td>1.4514</td>
<td>0.012</td>
</tr>
<tr>
<td>2.85±5.73</td>
<td>0.9119</td>
<td>-0.4453</td>
</tr>
</tbody>
</table>

Figure 10. Eigenvalues locus for model 2.

Figure 11. Voltage collapse induced by a saddle node bifurcation.

Figure 12. Time response of $E, \delta, s, \delta_L, V_L$ in the base case.

Figure 13. Locus of eigenvalues in model 3.
7. REFERENCES


Figure 14. Voltage collapse induced by a saddle node bifurcation.

8. APPENDIX

The state values of matrix $F_x$, $F_y$, $G_x$ and $G_y$ are:

\[
\begin{align*}
\mathbf{k}_1 &= \frac{-V(1)}{E} \left[ \frac{X_s - X'_s}{X_s} \cos(\delta - \alpha(1)) \right] - \frac{\alpha g V_x \sin(\delta - \theta_f)}{E} \\
\mathbf{k}_2 &= \frac{V(1)}{E} \left[ \frac{X_s - X'_s}{X_s} \sin(\delta - \alpha(1)) \right] - \frac{\alpha g V_y \cos(\delta - \theta_f)}{E^2} \\
\mathbf{k}_3 &= \frac{-V(1)}{E} \left[ \frac{X_s - X'_s}{X_s} \cos(\delta - \alpha(1)) \right] + \frac{\alpha g V_x \cos(\delta - \theta_f)}{E} \\
\mathbf{k}_4 &= \frac{V(1)}{E} \left[ \frac{X_s - X'_s}{X_s} \sin(\delta - \alpha(1)) \right]
\end{align*}
\]

For a saddle node bifurcation.