Dynamic stability analysis of tensegrity systems

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Abstract

The present paper discusses the possible existence of elastic dynamic instabilities appearing during non-linear dynamic response of tensegrity systems such as tripod simplex and Geiger dome. A geometric non-linear dynamic procedure had been implemented to solve the equations of motion using a step by step numerical integration method and a modified Newton–Raphson scheme. A criterion in the form of a degree of stability derived as a function of the time evolution of the variation in strain energy and complementary strain energy is used for the investigation. The effect of active control on the stability of the structure is also studied. Active damping is successfully added to the structure using collocated displacement actuator–force sensor pairs located on the lower end of each cable and a robust control strategy based on decentralised collocated integral force feedback. It has been shown that the instability problem of tensegrity systems may be solved using active control.

Keywords: Tensegrity, non-linear, dynamic, stability, vibration control, IFF

1. Introduction

“Tensegrity” systems are freestanding space pin-jointed structures made up of a network of continuous tensile elements and “discontinuous” compressive components. A tension member is referred to as a cable (wire or tendon) and a compressive member as a strut (rod or bar). A general definition of these systems is given by Motro and Bouderdala (1996):

Tensegrity systems are space reticulated systems rigidified by a state of self-stress. Members are rectilinear and are of equivalent size. Compressive members (struts) constitute a discontinuous set, tensile members (cables) a continuous set. Every node receives at least one strut and three cables.
Even though struts have bilateral rigidities, they are always in a compressive state for Tensegrity systems.

The static and dynamic analysis of tensegrity structures had been intensively investigated by researchers. Sultan provided in his review (Sultan, 2006) an introduction to the evolution of tensegrity research. Relevant reviews on static and dynamic analysis are presented by Hernandez and Mirats Tur (2007) and Mirats Tur and Hernandez (2009). Tensegrity structures may become unstable due to vibrations induced by wind, traffic, waves or even earthquakes which may cause the failure of the system. The instability phenomena is the disproportional amplification of the response (e.g. displacement, deformation, stress) due to small or large excitations. In tensegrity systems, both material and geometric instabilities can occur under dynamic excitations which lead to local and global instabilities: yield or failure of materials, buckling of bars, slacken of cables and snap through effect.

Only few papers have focused on the stability analysis of tensegrity structures. Connelly and Terrell (1995) introduced a stability criterion called super stability and studied some prismatic systems. A super stable tensegrity structure is guaranteed to be stable, for any level of self-stress and material properties, as long as every member has a positive rest-length. Lazopoulos (2005) studied the global instability of a T-3 tensegrity system and the local-Euler-buckling of the struts. He derived the critical conditions and described the post-critical behaviour. Ohsaki and Zhang (2006) presented the stability conditions of tensegrity structures based on the eigenvalues and eigenvectors of the linear and geometrical stiffness matrices. Recently, Amendola et al. (2014) performed the experimental investigation of the softening--stiffening response of tensegrity prisms under compressive loading. They showed that the compressive response of tensegrity prisms switches from stiffening to softening under large displacements, in dependence on the current values of suitable geometric and prestress variables. Böhm, Sumi, Kaufhold, and Zimmerman (2016) introduced an algorithm based on the repeated use of a form-finding procedure, using the static finite-element-method to identify compliant tensegrity mechanisms with multiple states of self-equilibrium. Zhang, Zhang, Feng, and Gao (2016) studied numerically and experimentally the snapping instability behaviour of tensegrity structures under torsion. Michielsen, Fey, and Nijmeijer (2012) proposed two parameter continuation diagrams of the loci of cyclic fold and period doubling bifurcations of periodic responses to assess upper bounds for the harmonic excitation amplitude in order to prevent dynamic instability of the tensegrity structure. Dynamic instability may be also prevented using active control.

An active control algorithm for vibration damping of spatial tensegrity structures was developed by Djouradi, Motro, Pons, and Crosnier (1998). De Jager and Skelton (2001) investigated the vibration control of a three unit planar tensegrity structure. Averseng and Crosnier (2004) proposed a vibration control approach based on robust control strategy. Their results were validated experimentally on a tensegrity plane grid of 20 m² where an actuation system is connected to the supports. Chan, Arbelaez, Bossens, and Skelton (2004a) performed experimentally the active control of a three stage tensegrity tower using local integral force feedback (IFF) control and acceleration feedback control. Xiao, Miao, and Chen (2011) investigated the active vibration control of a cable dome of Levy type under wind excitation. They established the active control formula based on the instantaneous optimal control algorithm. El Ouni and Kahla (2014) performed the active tendon control of a cable dome of Geiger's type using decentralised collocated IFF. Raja and Narayanan
Many other control strategies were used to control vibrations of tensegrity systems: e.g. optimal control theory (Raja & Narayanan, 2007), multi-objective control strategy (Ali & Smith, 2010), μ-synthesis controller (Li & Ma, 2011), H∞ approach (Amouri, Averseng, & Dubé, 2013).

In this paper a new criterion is proposed to investigate the dynamic stability of tensegrity systems. It consists in a degree of stability derived as a function of the time evolution of the variation in strain energy and complementary strain energy is used for the investigation.

The main objective of this paper is the discussion of the possible existence of elastic dynamic instabilities associated with cable slackening appearing during non-linear dynamic response of tensegrity systems, such as tripod simplex and Geiger domes. The effect of active control on the stability of the structure is also studied. Active damping is successfully added to the structure using collocated displacement actuator–force sensor pairs located on the lower end of each cable and a robust control strategy based on decentralised collocated IFF.

This paper is organised as follows: in Section 2, the modelling of the tensegrity structure and the stability criterion are presented. Furthermore, numerical examples are given in Section 3. Finally, in Section 4, some conclusions are drawn.

2. Dynamic stability

2.1. Uncontrolled system

Non-linear space truss elements are used to model the components of Tensegrity systems since they are made up of struts and cables. Let \((X, Y, Z)\) be a global coordinate system in which a space truss element, connected at both ends to node 1 and 2. A local element coordinate system \((x, y, z)\) is chosen such that \(x\) is along the element axis. The nodal displacement vectors, respectively, of node 1 and 2 are defined as \([u_1, v_1, w_1]^T\) and \([u_2, v_2, w_2]^T\). Considering the quadratic terms (representing the non-linear strain) in the strain–displacement equation, The space truss element stiffness matrix (Blandford, 1996; Leu & Yang, 1990), is written as

\[
[k] = \begin{bmatrix}
[k_0] & 0 & 0 \\
0 & [k_0] & 0 \\
0 & 0 & [k_0]
\end{bmatrix}
\]

where \([k_0]\) is the element elastic stiffness matrix, \([k_0]\) is the element geometric stiffness matrix and \([k_1], [k_2], [k_3]\) are the higher order nonlinear element stiffness matrices.

\[
[k_0] = E A L^2
\]

\[
[k_1] = E A (L f)^2
\]

\[
[k_2] = E A (L f)^3
\]

\[
[k_3] = E A (L f)^4
\]

\(E\) being the material modulus of elasticity, \(A, L\) and \(f\) are, respectively, the element cross section area, length and axial force, \(\Delta u = u_2 - u_1, \Delta v = v_2 - v_1\) and \(\Delta w = w_2 - w_1\).
a load dependent and time varying geometric configuration of the tensegrity system. Thus the evolution of the shape could pass through unstable geometries of the structure.

The incremental equation of motion governing the dynamic response of an uncontrolled tensegrity system subjected to dynamic loading written with respect to its configuration at time \( t \) using an updated Lagrangian formulation is:

\[
(7)
\]

with

\[
(8)
\]

where, \( M \), \( C \) and \( K \) are the mass, damping and stiffness matrices; \( \Delta u \), \( \Delta v \) and \( \Delta w \) are, respectively, vectors of incremental nodal acceleration, velocity and displacement; \( F_e \) is the external load vector, \( F_i \) is the system internal balanced force vector, \( F_r \) the residual force vector and \( \Delta t \) is the time increment.

Equation (7) is integrated in time by the Newmark constant-average acceleration method (Trapezoidal rule). The expressions for the incremental acceleration and velocity vectors are:

\[
(9)
\]

\[
(10)
\]

With

\[
(11)
\]

Combining Equations (9) and (10) with Equation (7), yields

\[
(12)
\]

Where \( \Delta K \) and \( \Delta F_i \) are, respectively, the effective stiffness matrix and the effective incremental load vector,

\[
(13)
\]

\[
(14)
\]

A modified Newton–Raphson iterative scheme is used in order to balance any residual forces within each time step, when adopting the initial stiffness formulation, Equation (7) becomes:

\[
(15)
\]

Since \( \Delta K \) is formed only one time at the start of the iterative process within each time increment \( \Delta t \) and that the solution procedure considers updating the nodal displacement vector, the incremental nodal displacement vector is set equal to the null vector at the beginning of the iterative process within each time increment. Consequently, the terms \( \Delta u \), \( \Delta v \) and \( \Delta w \) in Equations (4)–(6) are equal to zero at the instant \( t \) and iteration 0 and Equation (1) is then reduced to

\[
(16)
\]
in which \( \mathbf{d} \) is the element incremental end displacement vector.

The convergence within each time step is achieved when the following relation is satisfied (Cook, Malkus, & Plesha, 1989).

\[
\text{(18)}
\]

where to// is the pre-set tolerance.

The strain energy variation is expressed as follows

\[
\text{(19)}
\]

In which \( \mathbf{C} \) is the secant stiffness matrix.

The complementary strain energy variation is expressed as follows

\[
\text{(20)}
\]

Combining Equations (19) and (20), leads to

\[
\text{(21)}
\]

Which is the equation of a straight line passing through the origin of the axes \( \Delta U \) and \( \Delta U^* \) deviating by an angle of

\[
\text{(22)}
\]

from the abscissa axis \( \Delta U \). This angle is constant and equal to \( \pi/4 \) for stable linear elastic systems. For non-linear stable structures this angle lies in the intervals \([0, \pi/2]\) and \([\pi, 3\pi/2]\), whereas for systems which may display instabilities, the angle would also appear in the intervals \([\pi/2, \pi]\) and \([3\pi/2, 2\pi]\). By denoting \( \beta_{t+\Delta t} = \theta_{t+\Delta t} - \pi/4 \), He, Chen, Dong, and Wang (2003) introduced a degree of stability defined as

\[
\text{(23)}
\]

If \( DS_{t+\Delta t} \geq 0 \), the structure is stable; when \( DS_{t+\Delta t} < 0 \), the structure is unstable.

**2.2. Controlled system**

In order to study the effect of active control on the stability of the tensegrity structure, a control strategy using collocated displacement actuator-force sensor pairs located on the lower end of each cable and a robust control strategy based on decentralised collocated IFF, is employed. By measuring the force change \( T \) in the active element and controlling the active displacement, the amplitude of vibration can be reduced. The IFF controller always absorbs energy from the structure and hence adds damping. Note that this control is decentralised since every sensor signal is feedback to the corresponding collocated actuator. Figure 1 explains the proposed control strategy.

Figure 1. Active element equipped with an IFF.