

Reliability Analysis of El Houareb Dam Kairouan – Tunisia – Case Study

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(Received 27/7/2010; accepted for publication 24/2/2011)

Abstract. The present paper deals with the reliability analysis of El Houareb earth dam (Tunisia). A conventional Three-dimensional limit equilibrium analysis has been performed for the dam using CLARA computer program. Also a simplified reliability associated with particular failure mechanism, is considered. The Spatial variability of soil properties is represented by a random field. The global probability of failure of El Houareb earth dam has been investigated.

Keywords: Earth Dam, Reliability Analysis, Three-dimensional Limit Equilibrium Analysis

1. Introduction

Recently progress has been made in the application of probabilistic approach to geotechnical engineering problems. It appears that the probabilistic techniques offer the only systematic way of treating and reducing uncertainty within the design process. Moreover, it is only in terms of probability that the degree of uncertainty can be related quantitatively to the reliability of the engineering and geotechnical design of earth dam [1]. The correlation of mechanical and physical properties between two points in space within a dam exists [2].

Risk assessment of embankment dams must address both consequence of dam's slope failure and hazard or probability of failure (or reliability index), both require an understanding of the failure mechanism in order that the probabilities can be addressed.

2. Reliability Analysis of Earth Dams

Geotechnical engineering reliability analysis is concerned with finding the reliability or probability of failure (or reliability index) of structure or system. The benefit of reliability analysis in geotechnical engineering can be summarized in the few following points:

- to highlight the uncertainties in design of these structures. Reliability analysis plays a major role in considering the uncertainties influencing the design of earth structures. For example, an optimum procedure for design of an embankment can be discussed where there are uncertainties with regard to a stability problem.
- allow the geotechnical engineer to quantify the effect of various failure preventive measures on these structures in order to develop both, an inspection and maintenance programs.

The capacity-demand is the simplest and most utilized model for the reliability evaluation of most geotechnical structures, in particular existing earth dams. In this model, the question of interest is the probability of failure related to a load event rather than the probability of failure within a time interval. Reliability assessment methods are being adopted to develop the rigorous risk-management programs. Implementing the programs will ensure that the safety is maintained to a robust and acceptable level.

The reliability analysis typically includes the following steps:

- Establishing limit states,
- Identifying failure modes,
- Formulating limit state functions,
- Analyzing uncertainty,
- Evaluating reliability, and
- Assessment results.

The reliability index of earth dams is commonly taken as the value corresponding to the failure surface associated with minimum reliability index. The conventional factor of safety is defined as the ratio of limit capacity of soil to a demand in terms of loads, as follows:

$$F = \frac{R}{S} \quad (1)$$

in which:

R: The capacity (resisting force or resisting moment); and

S: The demand (driving force or driving moment).

In probabilistic modeling of safety, R and S are assumed to be random variables. Let $f_R(r)$ and $f_S(s)$ be the probability densities functions of variables R and S. The probabilistic measure of safety is the probability of failure, P_f in which should be smaller than certain reference values set a priori. The probability of failure is defined as (failure occurs if $R < S$):

$$P_f = P\left(\frac{R}{S} \leq 1\right) \quad (2)$$

Assuming statistical independence between the variables R and S, the probability of failure can be expressed as:

$$P_f = \int_{-\infty}^{+\infty} f_S(s) \left(\int_{-\infty}^{r=s} f_R(r) dr \right) ds \quad (3)$$

The use of formulation (3) of probability of failure makes the simplification possible only for certain types of distribution of R and S such a normal distribution. In such case the notion of safety margin, $MS=R-S$ (Cornell 1970) can be introduced. It is Possible to derive the density function $f_{MS}(MS)$ of the random variable MS and the risk of failure is given as:

$$P_f = \int_{-\infty}^0 f_{MS}(MS) d(ms) \quad (4)$$

In general, the calculus of the integrals in the preceding equation is particularly cumbersome. In this case, the safety is defined by the reliability index, β , as [3]:

$$\beta = \frac{E\{MS\}}{\sigma_{MS}} \quad (5)$$

in which:

$E\{MS\}$: Expected value of MS; and σ_{MS} : Standard deviation of MS, p

Equation (5) provides a simple quantitative basis for assessing the risk i.e. probability of failure.

The advantage of reliability index is that it can be determined from two first statistic moments (mean value and variance) of probability density functions of R and S without any assumption on the specific shape of these functions.

However, embankments are considered as systems composed of several “infinite” number of possible failure surfaces associated with different reliability indices. Therefore, the global probability of failure of embankment is however, at least for the moment, a complicated problem to handle since correlation exists between different failure surfaces. Subsequently, the global probability of failure will be investigated in the case study of El houareb dam.

3. Case Study: El Houareb Dam

3.1 Description and Presentation

El Houareb dam (Central Tunisia) is selected as a case study to perform the reliability analysis. The main objective of this dam is to contain Merguellil Wadi Floods. El Houareb reservoir is a man-made water-body built on the Oued Merguellil, 35 km east of Kairouan, for flood-control and water-supply purposes. Where the river emerges from the Dorsale, the reservoir is bordered by higher land, but also has extensive flat shores. It retains the waters which once flowed into Sebkha Kelbia (Kairouan city, Tunisia). The average reservoir depth is nearly 20 m, but in periods of poor rainfall, it can remain completely dry for several years on end. The water plant grows commonly in the reservoir and provides the main food-source for wild fowl. The hydraulic characteristics of El Houareb dam are:

- Area of basin pouring: 1,120 km²
- Total capacity: 95.3 x 10⁶ m³
- Yearly average contribution: 70 x 10⁶ m³.

El Houareb dam is founded on a sedimentary basin. It is filled with a Triassic, Cretaceous; Tertiary aged marine and fluvial sedimentary rocks fractured rock mass on its right side. The dam has a height of 32 m and crest width of 8.5 m. A berm of 40 m long located at the downstream side. The clay core, which provides impermeable barrier within the body of the dam, has a sloppy upstream, 6 m wide at the top and 21 m wide at the foundation level. The compacted materials, Table (1), were evaluated according to their maximum dry unit weight (γ_{dmax}), optimum water content (w_{opt}), specific gravity (G_s), liquid limit (LL) and plasticity index (I_p), all the parameters except specific gravity indicated the desirable characteristic as an impervious fill material. Engineering analyses for the proposed dam were performed to evaluate a suitable dam section for the site conditions and available on-site construction materials, Figure (1).

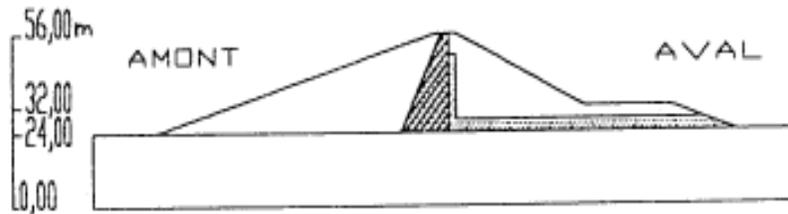


Fig. (1). Cross-section of El Houareb dam, Elevation is in meters

Table (1). Geotechnical properties of soils and earth dam materials. γ =dry density, c = cohesive strength, Φ' =friction angle, E = Young's modulus, ν = Poisson ratio

Nature of soils	γ (KN/m ³)	c (Mpa)	Φ' (°)	E (Mpa)	ν (-)
Clay	20	0.07	20	30	0.30
Sand	20	0.08	20	20	0.30
Filter	20	0.09	30	20	0.30
Clay Hard Core	20	0.08	25	50	0.30
Embankment compacted	25	1.00	40	100	0.25

3.2 Equilibrium Limit Analysis

The program CLARA, 2D/3D [4] may be used to conduct the Limit Equilibrium slope stability analysis in two or three dimensions. This program carries out the analysis of soil or rock slope stability both in static and seismic states. Moreover, different modes of failure, including circular sliding surfaces, ellipsoids, wedges and compound surfaces, could be implemented in CLARA. CLARA provides a choice of the method of analysis including the following:

- Fellenius's method,
- Bishop's method and uses 3D extensions of Bishop's Simplified,
- Spencer's method, and
- Janbu's method.

For the current study, the circular (Figure 2) and ellipsoid failure mechanisms have been considered:

Table (2). The Factor of safety corresponding to the different types of failure mechanism

FAILURE MECHANISM	Factor of Safety
Upstream slope failure 2D	1.6
Upstream slope failure 3D	2.01

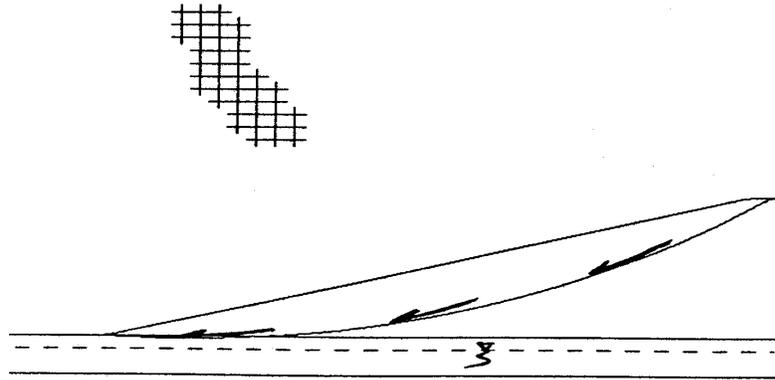


Fig. (2). Analysis of upstream slope of el Houareb dam using CLARA 2D to find the circular slip surface.

An example of CLARA 3D analyses of El Houareb dam is show in Figure (3). For the reliability analyses of El Houareb dam, only the ellipsoid failure mechanism will be considered.

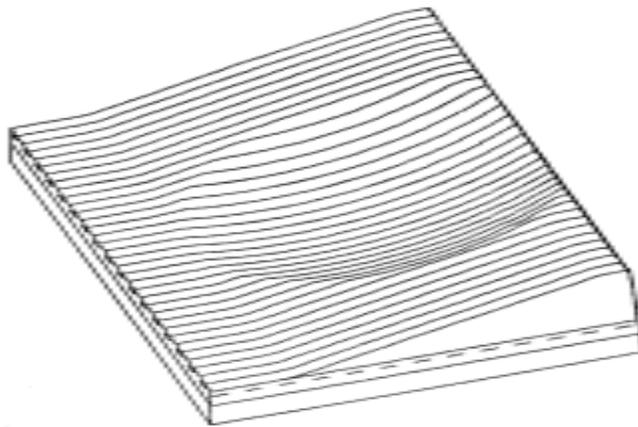


Fig. (3). Analysis of upstream slope of El Houareb dam using limit equilibrium to find the ellipsoid failure surface (3D extension of Bishop's simplified method, Clara 3D)

4. Reliability Assessment

4.1 Auto-Correlation Function

Spatial correlation has long been ignored in modeling variability of soil properties. However, the spatial dependency within the medium should be considered,

particularly in a strongly compacted soil such as earth dams [5]. To take into account spatial correlation, it is possible to model the spatial variability of soil properties with a spatial stochastic process also known as random field [6]. In this process, the variable exhibits auto-correlation, the tendency for values of the variable at one point to be correlated to values at nearby points. Recently, special attention has been given to the role of spatial correlation. Some recent papers dealing with this concept include those by Mrabet and Giles [7].

Many studies stressed out the effect of existing auto-correlation on the results of probabilistic models of compacted earth slopes analysis. Ignoring auto-correlation is conservative and considerably more than desired [8]. The analysis that considers the typical auto-correlation distances results in reduction of probability of failure [9].

Extensive measurement program performed on earth dams such Mirgenbach and Vieux-Pre dams [10] and others led to the conclusion that, for such structures, a significant spatial correlation exists. The influence distance at which auto-covariance becomes negligible is sensitive to the construction procedure as well as the material nature, but is practically identical for all properties. A pronounced anisotropy of the auto-covariance exists; the vertical distance of influence is of the order of meters, while the horizontal distance of influence is of the order of tenth of meters [7, 8]. Similar pattern has been found for mechanical properties and the exponential auto-correlation function for two different points within the compacted soil of the El Houareb dam has been retained.

Due to the lack of data concerning the horizontal auto-correlation function, Anderson's work (1981) is utilized to establish one. Therefore, the following function is obtained:

$$\rho_{hor9x} = \exp(-0,065x) \quad (6)$$

4.2 Global Probability of Failure

The reliability index of earth dam is commonly taken as the value corresponding to the failure surface associated with minimum reliability index. However, embankment dams are investigated as systems composed of several infinite number of possible failure surfaces associated with different reliability indices. Therefore, the global probability of failure of an embankment dam is however, at least for the moment, a complicated problem to handle since correlation exists between different failure surfaces. In practice, the reliability of the whole system may be governed by a few subsystems or components. The calculated probability associated to the critical failure surface constitutes a lower limit of the global probability of failure of El Houareb dam. Subsequently, we calculate the global probability of failure in respect to the following conditions:

- Ceros-section of El Houareb dam as considered in the above analysis (figure 1)
- Horizontal auto-correlation distance =60 m
- Vertical auto-correlation distance =3.1 m

- Coefficient of variation of the cohesion of the El Houareb dam= 0.35

The dam is divided into a series of ellipsoids, which are vertically independents and separated by 3 m.

Table (3): Failure Probability

H_i (m)	F_i	β_i	P_{fi}
0 m			
Contact	2.01	3.45	0.025
Dam-Foundation			
3	2.63	4.00	0.00013
6	2.95	4.23	0.0003
9	3.50	4.55	0.00002
12	3.78	4.80	0.000003
15	5.14	5.23	0.0000023
18	7.02	5.40	0.0000001
21	9.04	5.63	0.000000025
24	15.02	5.90	0.000000004
27	18.3	6.13	0.0000000032
30	25	6.23	0.0000000005

H_i = depth of the ellipsoid regarding the interface dam-foundation (or the bottom of the dam)

F_i = Factor of Safety corresponding to the ellipsoid failure Surface number i.

β_i = Minimum Reliability index corresponding to the ellipsoid failure surface number i.

P_{fi} = Failure probability corresponding to the ellipsoid failure surface i,

The global failure probability is calculated using the following equation:

$$P_{global} = 1 - \prod_{i=1}^n (1 - P_{fi}) \quad (7)$$

Which is found to be = 0.02. This value is close to the value associated with the critical ellipsoid failure surface shown in Table 93). This calculation shows that the concept of global probability is coherent; and should be considered, later, as the global probability of the project.

In reality, these results show that other sources of uncertainty should be taken into account including, but not limited to:

- Mechanisms of failure
- Properties of materials
- Entire history of dam behavior
- Ignorance of the auto-correlation function.

5. Conclusion

The rational way to managing uncertainty in evaluating the safety and reliability of El Houareb dam is to reduce it. The reliability analysis should include all sources of uncertainty.

Similar reliability analysis could be performed using conditional random field to evaluate the uncertainty related to spatial variation of the material properties within the dam based on quality control.

6. References

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التحليل الإحصائي لسد الهوارب بوسط الجمهورية التونسية

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(قدم للنشر في ٢٧/٧/٢٠١٠، وقبل للنشر في ٢٤/٢/٢٠١١)

ملخص البحث. هذا البحث يهتم بالتحليل الإحصائي لسد الهوارب بوسط الجمهورية التونسية. لأجل القيام بالتحليل التوازن الحدي ثاني و ثالث الأبعاد تم الاعتماد على برنامج كلارا (CLARA). كذلك تم إبراز دور النموذج الإحصائي في فهم الانزلاق الأرضي في منحدرات الجانبيه للسد. للأخذ بعين الاعتبار المتغيرات المكانية لخصائص التربة تم إدراج الحقل العشوائي في هذا البحث. من خلال هذا البحث توصلنا إلى حساب إمكانية الانزلاق الأرضي للمنحدرات الجانبيه لهذا السد وفقا للتحليل الإحصائي العشوائي الممثل.